3 MIXED REVISION
CHAPTERS 7 • 8 • 9

Multiple choice

1 Which one of the following is not a prime number?

A 1 B 2 C 137 D 211 E 503

2 Matrix $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$ and matrix $\mathbf{B} = \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}$. Matrix $\mathbf{A} + \mathbf{B} = ?$

A $\begin{bmatrix} 1 & 3 & 5 & 1 \\ 4 & 0 & 3 & 2 \end{bmatrix}$ B $\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 5 & 1 \\ 3 & 2 \end{bmatrix}$ C $\begin{bmatrix} 6 & 4 \\ 7 & 2 \end{bmatrix}$ D $\begin{bmatrix} 14 & 7 \\ 20 & 4 \end{bmatrix}$ E $\begin{bmatrix} 9 & 15 \\ 11 & 9 \end{bmatrix}$

3 $\cot\left(\frac{5\pi}{6}\right) =$

A $\frac{1}{\sqrt{3}}$ B $\frac{1}{\sqrt{2}}$ C $\sqrt{3}$ D $-\sqrt{3}$ E -1

4 $3.\overline{142857}$ is

A equal to π B an irrational number C an integer

D equal to $\frac{22}{7}$ E a transcendental number.

5 The matrix $\mathbf{A}$ has order $2 \times m$, matrix $\mathbf{B}$ has order $4 \times n$ and matrix $\mathbf{AB}$ has order 2×3. The values of m and n are:

A $m = 3, n = 3$ B $m = 2, n = 3$ C $m = 4, n = 3$

D $m = 2, n = 4$ E $m = 3, n = 4$

6 $\sin\left(\frac{\pi}{12}\right)$ can be expressed as

A $2\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)$ B $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

C $\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$ D $\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

E $\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

7 Which one of the following sets is **not** closed under multiplication?

A Real numbers B Irrational numbers C $\{-1, 0, 1\}$

D Integers E Positive integers

8 For the matrix $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $\mathbf{A}^3 = ?$

A $\begin{bmatrix} 6 & 9 \\ 12 & 15 \end{bmatrix}$ B $\begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix}$ C $\begin{bmatrix} 35 & 152 \\ 189 & 152 \end{bmatrix}$

D $\begin{bmatrix} 8 & 27 \\ 64 & 125 \end{bmatrix}$ E $\begin{bmatrix} 116 & 153 \\ 204 & 269 \end{bmatrix}$

9780170250276

9 Expressed as a difference, $\sin(x)\sin(5x) =$

A $\frac{1}{2}[\cos(4x)-\cos(6x)]$ B $\frac{1}{2}[\cos(4x)+\cos(6x)]$ C $\frac{1}{2}[\cos(x)-\cos(5x)]$

D $\frac{1}{2}[\sin(4x)-\sin(6x)]$ E $\frac{1}{2}[\cos(6x)-\cos(4x)]$

Short answer

1 a Show that the equation $4x + 6y = 1987$ has no integer solutions.
b Find a counter example to show that the statement below is false.
The equation $3x + 5y = 40$ has no positive integer solutions.

2 Show that the matrix equation below has no solution. Justify your reasoning.

$$\begin{bmatrix} 12 & 28 \\ 15 & 35 \end{bmatrix}\mathbf{X} = \begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}$$

3 Select a suitable double angle formula to express $\sin(x)\sin(2x)$ in terms of $\cos(x)$ only.

4 Construct the decimal 0.05005000500005… by adding fractions of the form $\frac{1}{2\times 10^a}$, where $a \in \boldsymbol{Z}^+$.

5 Find the solution to the equation $\begin{bmatrix} 4 & 2 \\ 6 & 5 \end{bmatrix}\mathbf{X} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

6 a Expand the expression $\cos(x + 2x)$ using compound angle and double angle formulas.
b Hence show that $\cos(x + 2x) = \cos(x)[1 - 4\sin^2(x)]$

Application

1 Prove by contradiction that $x^3 + x - 7 = 0$ has no rational solutions.

2 a Show that the following statement is false.
If $2n + 7$ is odd, then n is odd, where $n \in \boldsymbol{Z}$.
b Show that for all $n \in \boldsymbol{Z}$, $2n + 7$ is odd.
c Show by mathematical induction that $\frac{1}{7\times 9}+\frac{1}{9\times 11}+\frac{1}{11\times 13}+\cdots$ to n terms $=\frac{n}{7(2n+7)}$, where $n \in \boldsymbol{N}$.
d For what values of n will the reciprocal of the sum in part **c** be an odd integer?

3 $\mathbf{C} = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$. Find the inverse matrix $\mathbf{C}^{-1}$.

4 Solve for **X**.

$$2\mathbf{X} + \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}\mathbf{X} - \begin{bmatrix} 2 & 12 \\ 20 & 12 \end{bmatrix} = \begin{bmatrix} -14 & -6 \\ 22 & -3 \end{bmatrix}$$

5 Prove that $[\operatorname{cosec}(x) + \cot(x)][\operatorname{cosec}(x) - \cot(x)] = 1$.

6 Prove that $\frac{1+\cot(\beta)}{\operatorname{cosec}(\beta)} - \cos(\beta) = \frac{\sec(\beta)}{\tan(\beta)+\cot(\beta)}$.